CONGRESSO

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INSTITUTO DOS ATUÁRIOS PORTUGUESES

A Probability of Ruin Approach to Optimize Pension Fund Investments

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About the speaker

- **Abraham Hernández –** *President, VITALIS*
  Past President of PBSS, Member of the Strategic Planning Committee and the Nominations Committee, International Actuarial Association.
  Co-founder "Millas para el Retiro"
  PhD Student at ISEG, Universidade de Lisboa.
  Proud father of ten children

- VITALIS is an International Pension Consultant and Pension Fund Manager

Introduction

- Pension funds & other retirement assets add to ca. $40T, or almost ½ of global GDP.
- Variation in asset values is arguably the most significant component in the determination of the probability of ruin of pension funds (Sherris, 1993).
- Since changes in interest rates and other economic variables affect both assets and the present value of liabilities, asset allocation decisions may be also linked to the volatility not only of the assets, but of the difference or ratio of assets vs (present value of) liabilities (ALM).
- ALM techniques typically rely on complex financial models solved numerically through simulations and assumptions about risk preferences (utility function) of the sponsor.
- Probability of ruin (introduced by Lundberg in 1903 for non-life insurance programs) has been used as a measure of solvency for pension plans by several authors: Khorasanee (1995), Battocchio et al. (2003), Devolder & Tassa (2016), etc.
- Hernández-Pacheco & Salgado (2015) introduced the probability of ruin as a direct method to optimize asset allocation without assumptions on the utility function of the sponsor.

A Defined Benefit Pension Plan

$$P = 1\% \times salary \times years\, of\, service$$

$$APV = \sum_{U_A} P_k\, a_{65}\, v^{65-x_k}\, {}_{65-x_k}p_{x_k} + \sum_{U_R} P_k\, a_{x_k}$$

$$APVS = \sum_{U_A} s\, a_{x:\overline{xr-x}|@r}$$

$$PBO = \sum_{U_A} \frac{h_k}{65 - x_k + h_k} P_k\, a_{65}\, v^{65-x_k}\, {}_{65-x_k}p_{x_k} + \sum_{U_R} P_k\, a_{x_k}$$

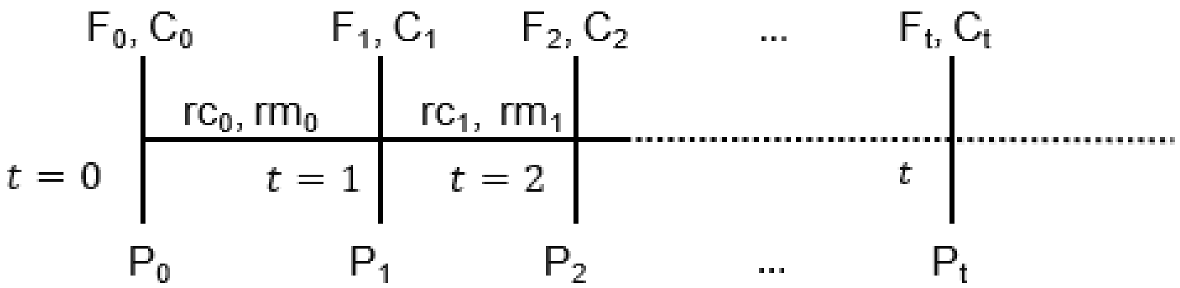
Contributions as a constant proportion of salaries:

$$\gamma = \frac{APV_{@r} - F_0}{APVS_{@r}} \qquad r > c$$

The PBO is calculated every year @ current rate of high quality long-term bonds, c.

The Fund (F_t)

- A proportion, α, of the fund is invested in the stock market and the rest in long-term corporate bonds:



- Then F_t is given by:

$$F_t = (F_{t-1} + C_{t-1} - P_{t-1}) \times (1 + \alpha rm_t + (1-\alpha)rc_t)$$

- $0 < \alpha \leq 1$ (no shorts)

A Defined Benefit Pension Plan

- Main concern is the Funding Ratio (FR_t):

$$FR_t = \frac{F_t}{PBO_t}$$

- If FR_t goes below certain level, the sponsor needs to make additional contributions or benefits need to be reduced
- Investing everything in bonds would not be a solution, since $r > c$

Incorporating ruin theory

- We will say that the plan faces ruin any time the Funding Ratio (FR_t) is less than β.
- Formally, we define T_α as:

$$T_\alpha = \min\{t \geq 1 : \ FR_t < \ \beta\}, \text{ for } t = 1, 2, 3, \ldots$$

$$\text{with } T_\alpha = \ \infty \text{ if } \ FR_t \geq \beta, \ \text{ for } t = 1, 2, 3, \ldots$$

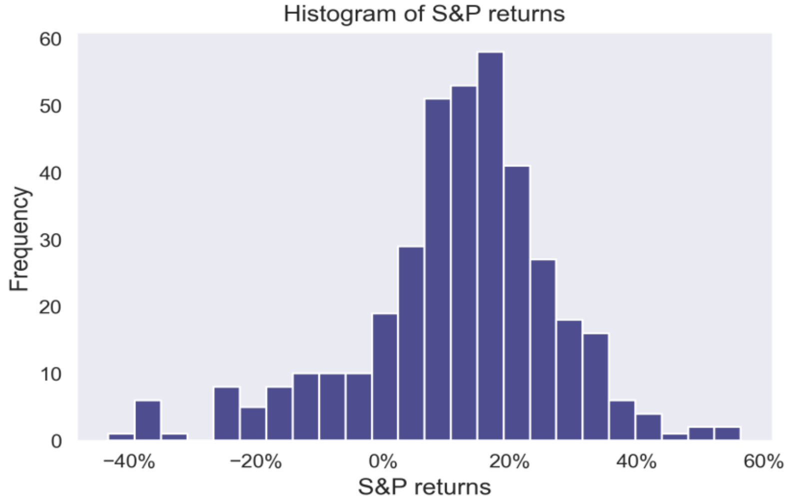
- The probability of ruin is then defined by:

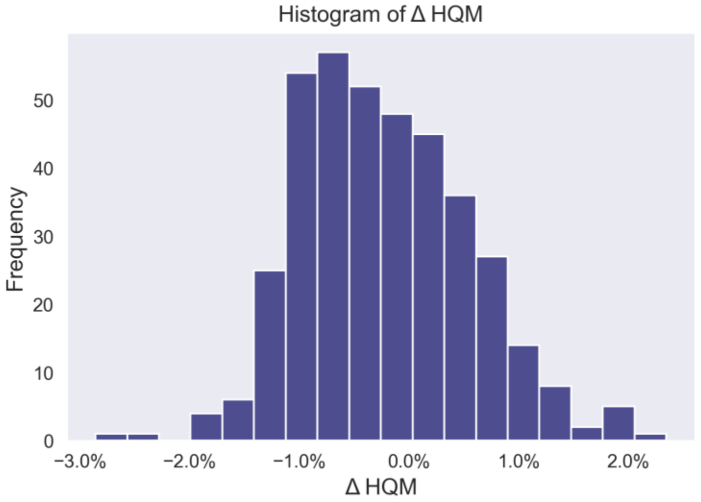
$$\varphi_\alpha = \Pr(T_\alpha < \ \infty)$$

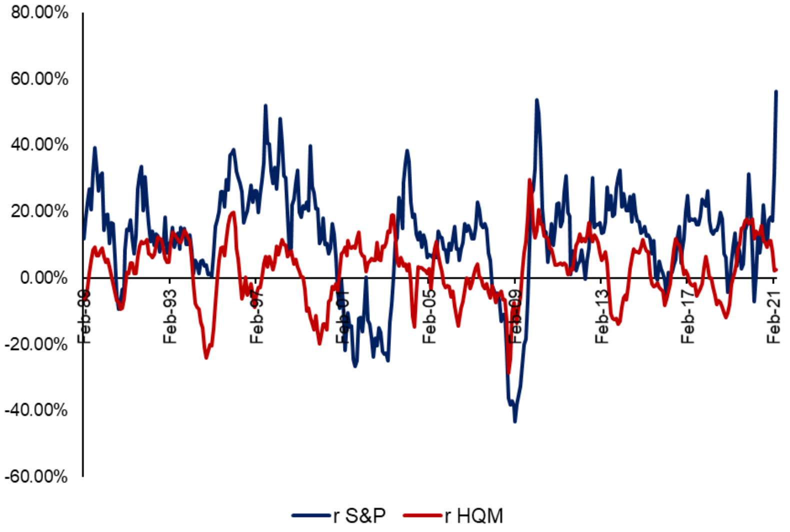
- Not the classical Cramér-Lundberg situation, but we can find through simulations the α that minimizes the probability of ruin.

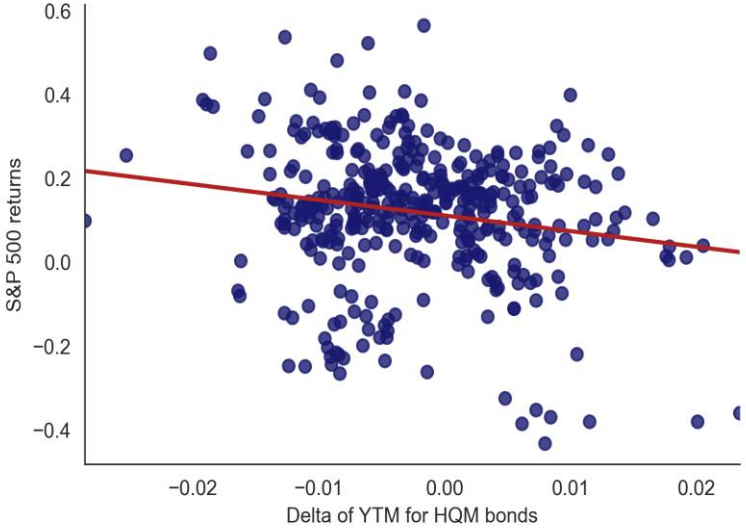
Example

- Initial group of 688 active participants and 26 retirees; and initial Funding Ratio of 73%; β=60%
- The figures to the right show the histogram, returns, and scatter plot of the S&P 500 and the annual increase in the yield of long-term corporate bonds (HQM).





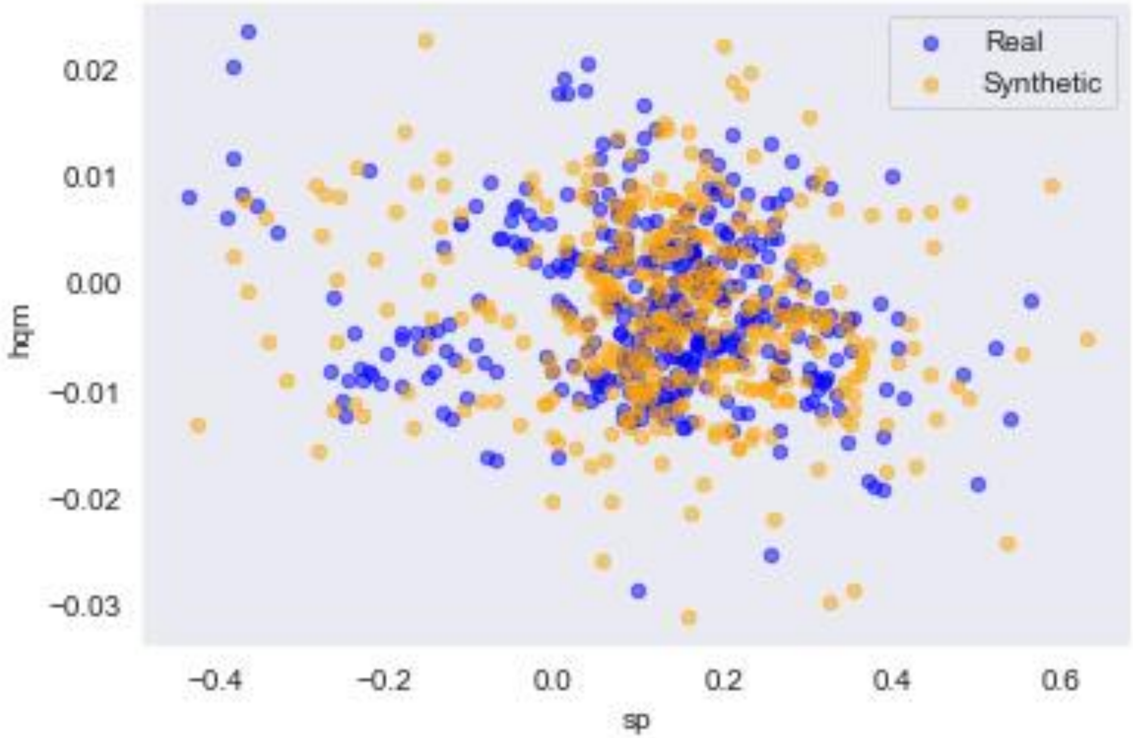




The Copula used to simulate r_t & Δc_t

- The series r_t & Δc_t were modelled with a Gaussian copula, with Kernel Distribution Estimates (KDE) for the marginals.

- The figure in the right shows a sample simulation using this copula and the KDE marginals versus the real scatter plot of the two series.



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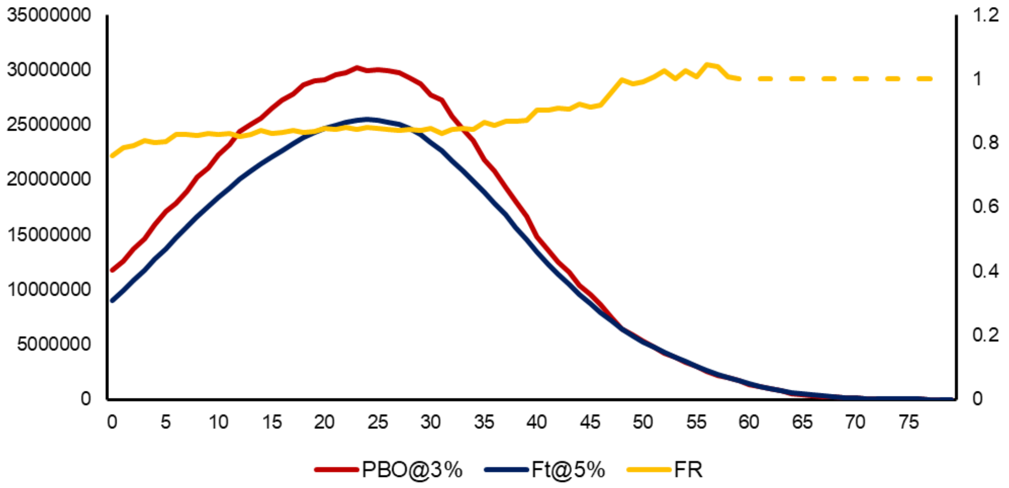
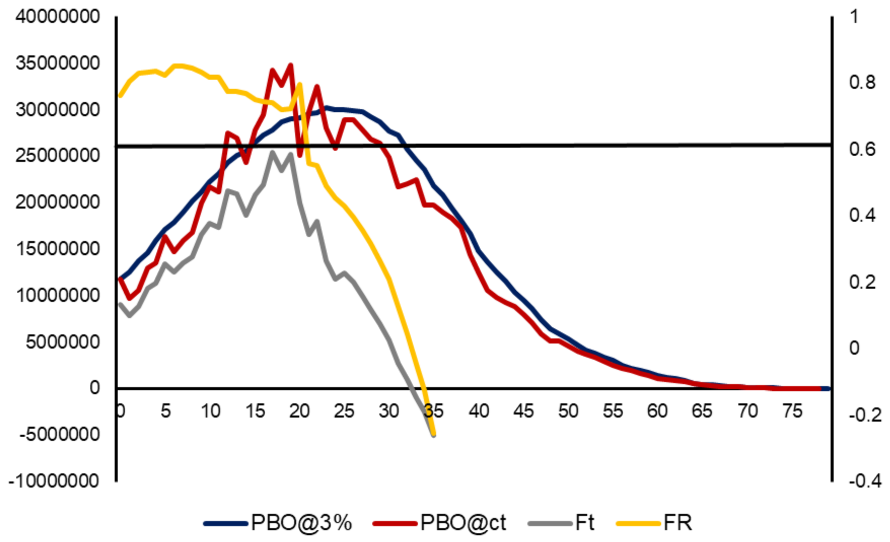
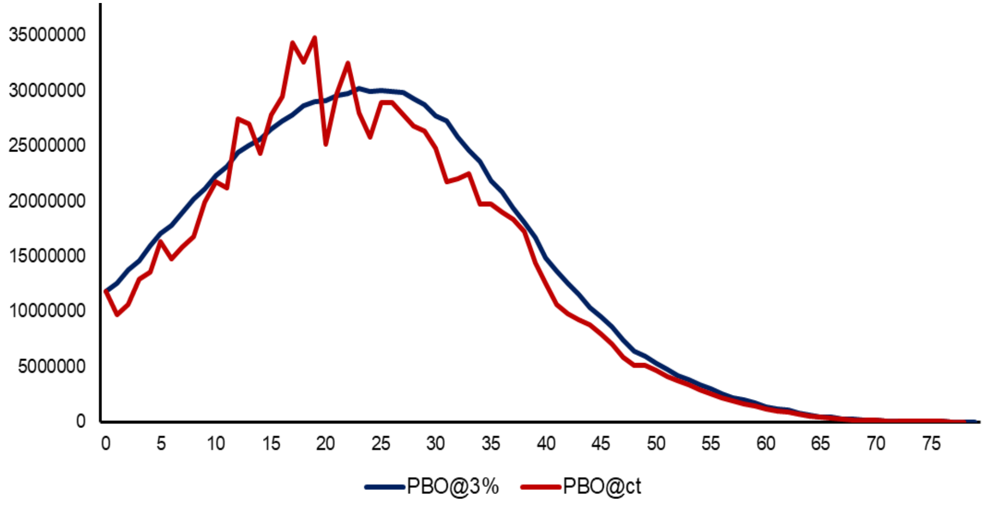
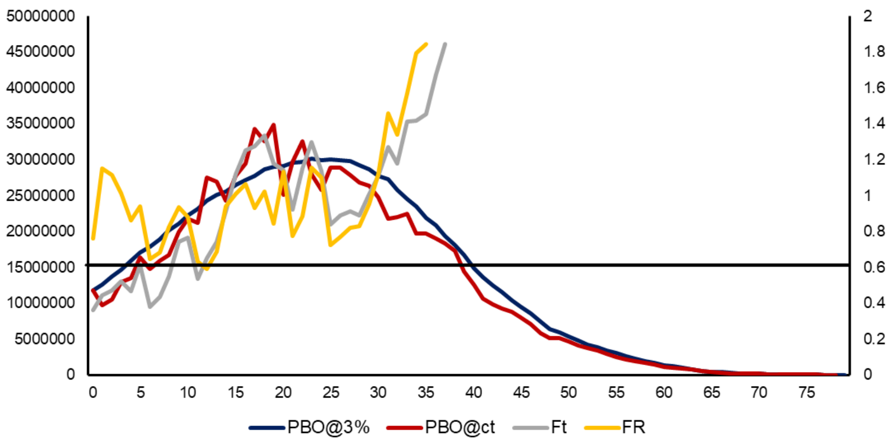
PBO@c_t and F_t with investments in stocks and bonds









A general outline of the simulation performed

- Simulate the survivorship of participants until the extinction of the group.
- Simulate r_t & ΔHQM from the copula and calculate FR_t for different values of α.
- The simulation can be summarized as follows:

```
for i to N (500 in this case):
     Simulate S&P returns and ΔHQM copula
     Build rm_t, rc_t, and c_t from these simulations
     Calculate the PBO for each year
     Calculate F_t and FR_t for each alpha (increments of 5%)
     Ruin occurs if FR_t <60% for any t
repeat for the next i
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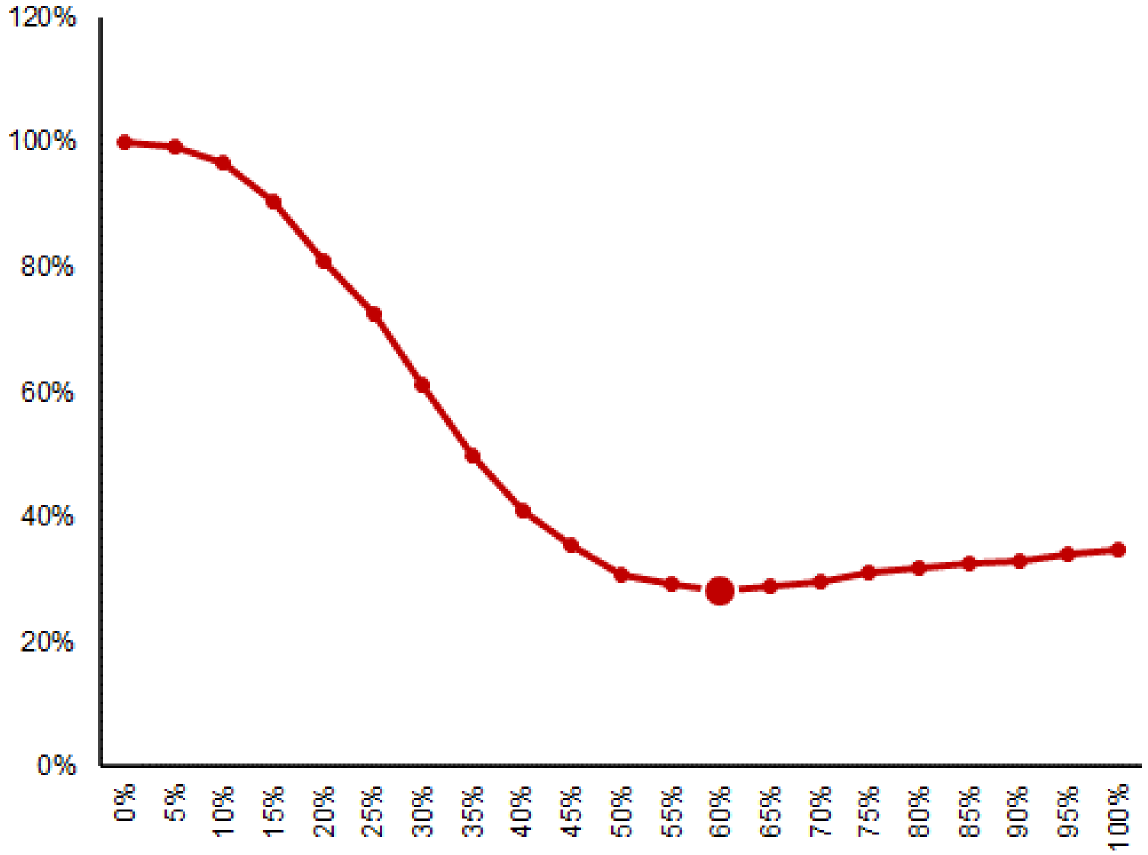
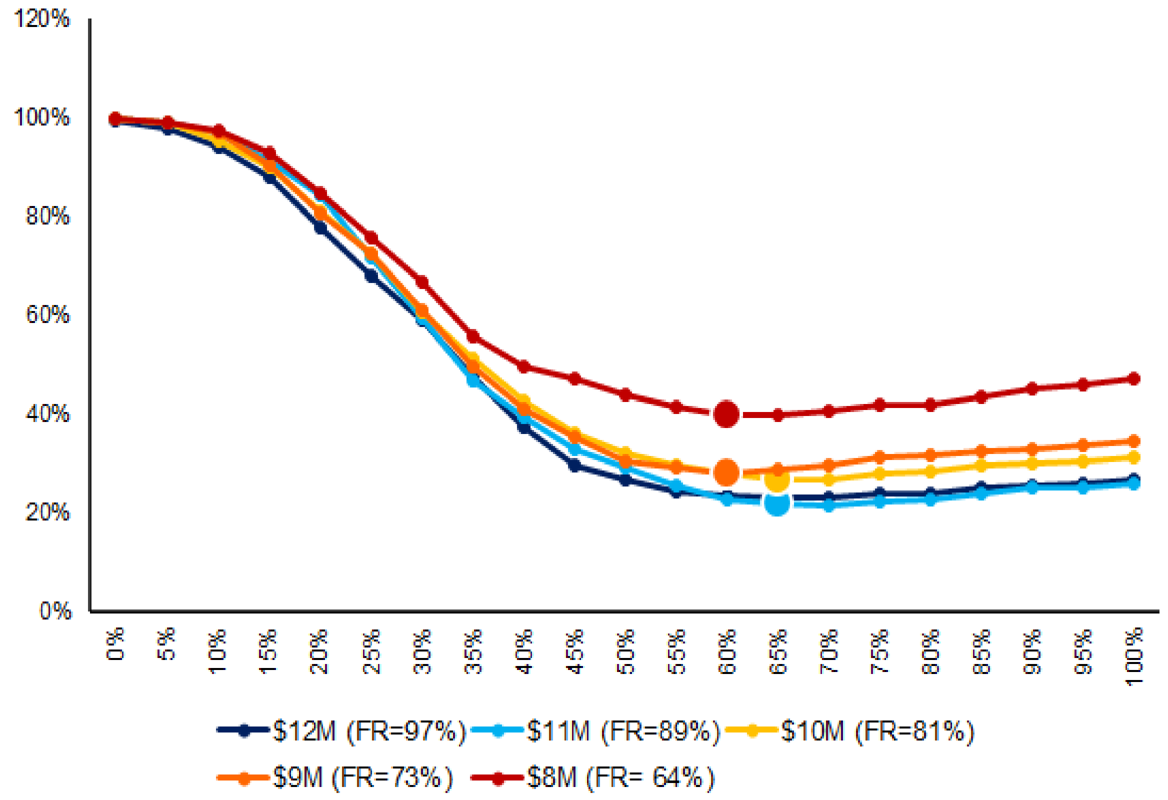
Estimated ruin probabilities for different levels of α



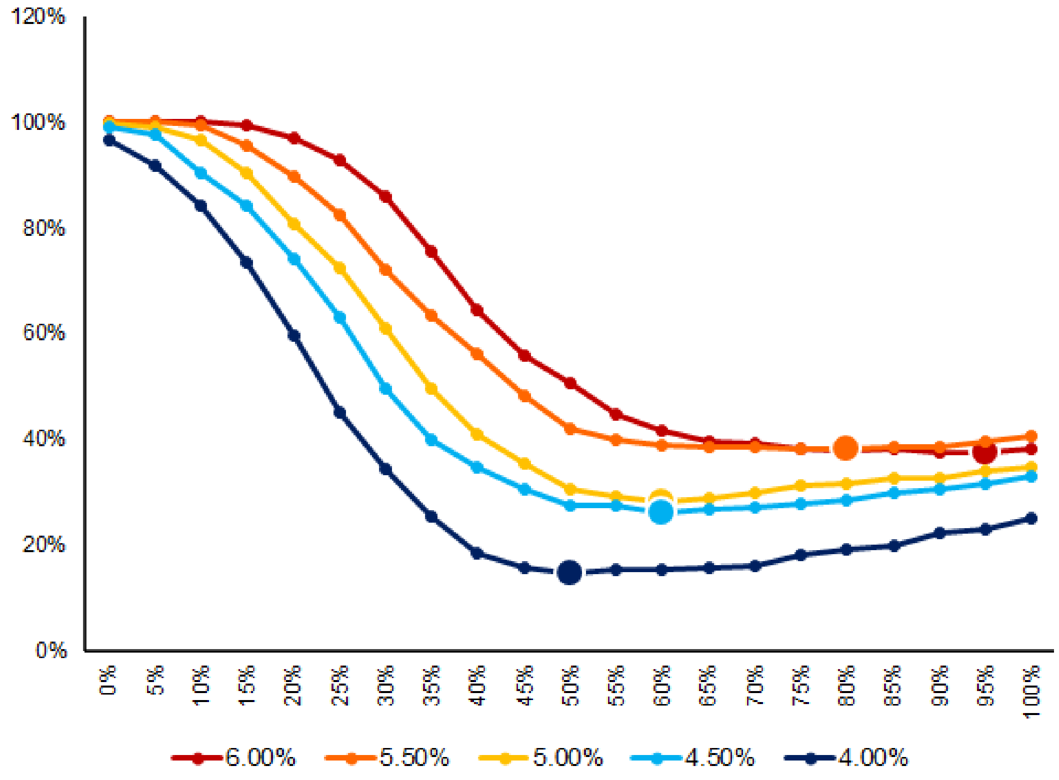
alpha	Ruin Probability
0	0.998
0.05	0.992
0.10	0.968
0.15	0.904
0.20	0.808
0.25	0.724
0.30	0.612
0.35	0.496
0.40	0.410
0.45	0.354
0.50	0.306
0.55	0.292
0.60	0.282
0.65	0.290
0.70	0.298
0.75	0.312
0.80	0.318
0.85	0.326
0.90	0.328
0.95	0.340
1	0.348

Ruin probabilities for different initial FR_t



alpha	$12M (FR=97%)	$11M (FR=89%)	$10M (FR=81%)	$9M (FR=73%)	$8M (FR= 64%)
0	0.996	1.000	1.000	0.998	1.000
0.05	0.980	0.992	0.990	0.992	0.992
0.10	0.942	0.968	0.954	0.968	0.974
0.15	0.882	0.912	0.900	0.904	0.928
0.20	0.778	0.842	0.810	0.808	0.848
0.25	0.681	0.718	0.723	0.724	0.758
0.30	0.591	0.596	0.607	0.612	0.670
0.35	0.475	0.470	0.513	0.496	0.560
0.40	0.377	0.396	0.429	0.410	0.498
0.45	0.299	0.330	0.363	0.354	0.474
0.50	0.269	0.292	0.321	0.306	0.442
0.55	0.244	0.258	0.299	0.292	0.414
0.60	0.236	0.230	0.281	0.282	0.400
0.65	0.230	0.218	0.271	0.290	0.400
0.70	0.230	0.214	0.269	0.298	0.406
0.75	0.238	0.224	0.283	0.312	0.418
0.80	0.238	0.226	0.287	0.318	0.418
0.85	0.251	0.242	0.299	0.326	0.436
0.90	0.257	0.252	0.301	0.328	0.452
0.95	0.263	0.254	0.305	0.340	0.462
1	0.271	0.260	0.315	0.348	0.472

Ruin probabilities for different assumed long term rate of return, r



alpha	6.00%	5.50%	5.00%	4.50%	4.00%
0	1.000	1.000	0.998	0.992	0.966
0.05	1.000	1.000	0.992	0.978	0.920
0.10	1.000	0.996	0.968	0.906	0.844
0.15	0.994	0.958	0.904	0.842	0.733
0.20	0.970	0.896	0.808	0.743	0.595
0.25	0.928	0.826	0.724	0.631	0.453
0.30	0.860	0.721	0.612	0.497	0.345
0.35	0.756	0.635	0.496	0.401	0.255
0.40	0.643	0.563	0.410	0.347	0.184
0.45	0.557	0.481	0.354	0.305	0.156
0.50	0.505	0.419	0.306	0.275	0.148
0.55	0.447	0.401	0.292	0.275	0.152
0.60	0.417	0.389	0.282	0.261	0.152
0.65	0.395	0.387	0.290	0.267	0.158
0.70	0.391	0.387	0.298	0.273	0.162
0.75	0.383	0.383	0.312	0.277	0.182
0.80	0.379	0.381	0.318	0.287	0.190
0.85	0.381	0.387	0.326	0.301	0.198
0.90	0.375	0.385	0.328	0.305	0.222
0.95	0.375	0.397	0.340	0.315	0.228
1	0.383	0.405	0.348	0.329	0.251

Further investigations

- Simulations with increases in salaries and pensions.
- Using stochastic mortality.
- Model more asset classes.
- Perform a dynamic optimization of α.
- Different values for β
- Use different models to simulate the financial series.

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Thank you!

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