CONGRESSO

Consistent Development Patterns

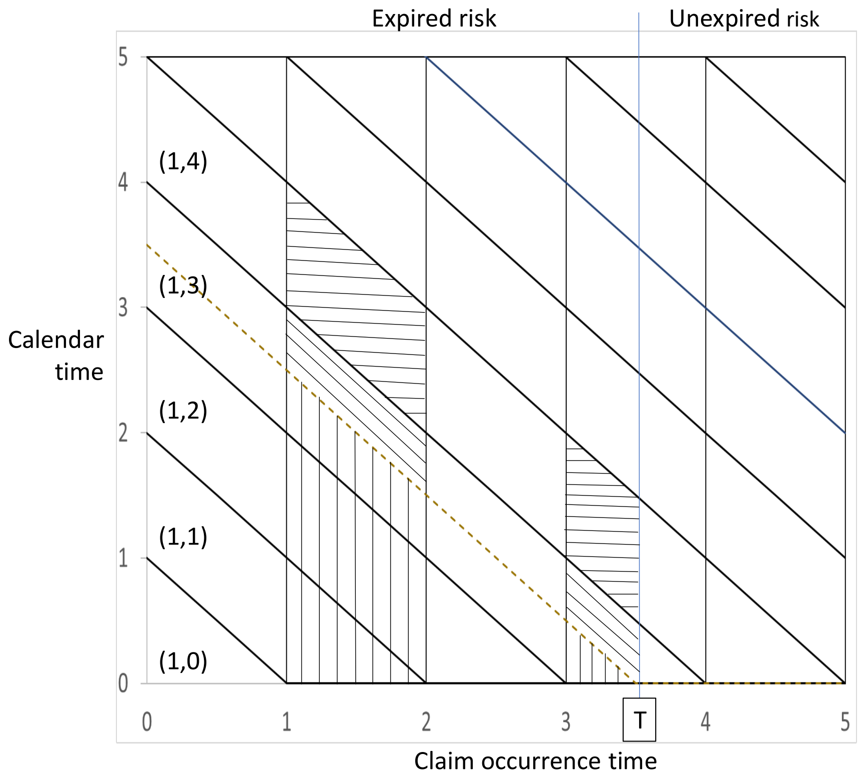
II. Discrete Claim Cohorts in Continuous Time

Walther Neuhaus

Problem definition

- The root cause: Traditional loss reserving uses discrete claim cohorts, to be valued at discrete points in time.
- The problem: what to do if we need a valuation at an unscheduled valuation date?
- The solution: modelling claim development in continuous time along the lines of Hesselager (1995).

Problem illustration



Outline of this presentation

1. Introduction
2. Framework model
3. Bornhuetter-Ferguson in continuous time
4. Unscheduled valuation
5. Calibration
6. Claim subcohorts
7. Other applications

Introduction

- Most textbooks describe loss reserving as an annual activity, performed on annual cohorts of claims.
- In practice, claim costs must be valued more frequently.
- Common approaches to intra-year valuations:
  - Interpolation of annual development patterns;
  - Exploding the time dimensions (e.g. years → quarters → months).
- None of these works in an unscheduled valuation ☹.

➢ My paper proposes a model that allows valuation at any point in continuous time, while retaining discrete cohorts.

Framework model – slide 1

For the sake of specificity, we consider claim numbers.

- Continuous time process $\{(T_n, U_n): n = 1,2, \cdots\}$.
- T_n are random claim occurrence times.
- U_n are random notification delays.
- T_n generated by Poisson process with intensity $W(t)\lambda(dt)$.
- U_n independent of T_n and i.i.d. $\sim F$.

➢ $Pr\{T_n + U_n \in (r_1, r_2] | T_n = t\} = F(r_2 - t) - F(r_1 - t)$.

Framework model – slide 2

Continuous time reporting pattern for a discrete claim occurrence cohort $(t_1, t_2]$:

- $E\left(N_{(t_1,t_2]}\right) = \int_{(t_1,t_2]} W(t)\lambda(dt) \; =: p_{(t_1,t_2]}$ (risk volume).
- $E\left(N_{(t_1,t_2],(r_1,r_2]}\right) = \int_{(t_1,t_2]} [F(r_2 - t) - F(r_1 - t)]W(t)\lambda(dt)$.

➢ $\pi_{(t_1,t_2],(r_1,r_2]} = \frac{\int_{(t_1,t_2]} [F(r_2-t)-F(r_1-t)]W(t)\lambda(dt)}{\int_{(t_1,t_2]} W(t)\lambda(dt)}$ (reporting pattern)

Bornhuetter-Ferguson in continuous time

Assume that recorded history starts at time $t = 0$ and that the current valuation date is $T > 0$. A predictor of the number of claims reported in $(r_1, r_2]$, for $r_1 > T$, is:

- $\bar{N}_{(t_1,t_2],(r_1,r_2]} = p_{(t_1,t_2]}\pi_{(t_1,t_2],(r_1,r_2]}\hat{\theta}_{(t_1,t_2]}$, with (for example)
- $\hat{\theta}_{(t_1,t_2]} = \dfrac{N_{(t_1,t_2],(0,T]}}{p_{(t_1,t_2]}\,\pi_{(t_1,t_2],(0,T]}}$ (grossing up as in chain ladder), or
- $\hat{\theta}_{(t_1,t_2]} = \dfrac{N_{(0,T],(0,T]}}{p_{(0,T]}\,\pi_{(0,T],(0,T]}}$ (averaging as in BF / Cape Cod), or
- A mix of the two, as in credibility formulas.

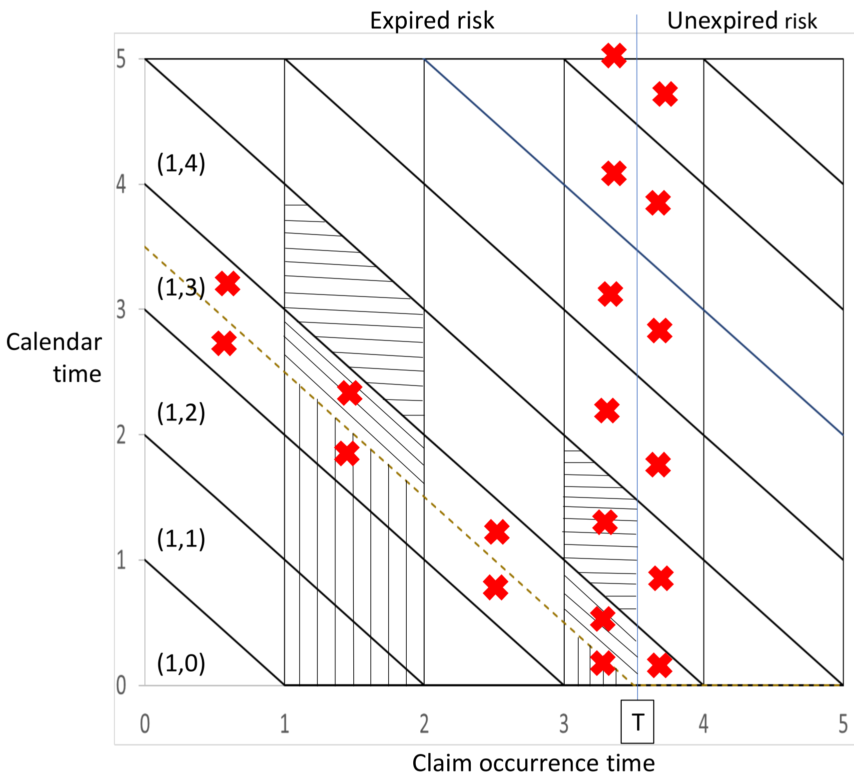
Interlude on prediction vs calibration

Often the tasks of calibration and prediction appear to be intermingled, when presented in textbooks. Here's my take:

- Prediction is predicting future claims, using a development pattern.
- Calibration means finding a suitable development pattern.
- Prediction is a transparent process and easily automated.
- Calibration often requires manual adjustments.
- Predictions are produced frequently, usually every quarter.
- Calibration should be done thoroughly but not too frequently.
- There is no compulsory nexus between the calibration method and the prediction method.

➢ This paper essentially proposes a new calibration method.

Unscheduled valuation – slide 1



The red crosses show all the development probabilities that the discrete time pattern cannot give us.

End of current period J=4
Valuation date T=3.5

Unscheduled valuation – slide 1

The "missing pieces" of claim development:

- Expired risk in past claim cohorts, as exemplified by claim cohort (1,2]: $\pi_{(1,2],(3,3.5]}$ and $\pi_{(1,2],(3.5,4]}$.

- Expired risk in the current claim occurrence period (3,3.5]: $\pi_{(3,3.5],(3,3.5]}$, $\pi_{(3,3.5],(3.5,4]}$ and the entire sequence $\pi_{(3,3.5],(e-1,e]}$ for $e > 4$.

- Unexpired risk in the current claim occurrence period (T,J]: $\pi_{(3.5,4],(3.5,4]}$ and the entire sequence $\pi_{(3.5,4],(e-1,e]}$ for $e > 4$.

- In addition, the actuary may need to split the exposure $p_{(3,4]}$ between $p_{(3,3.5]}$ and $p_{(3.5,4]}$ in another way than pro rata temporis.

Unscheduled valuation – slide 2

A super simple example of filling the gaps:

- We assume that $p_{(t_1,t_2]} = t_2 - t_1$.
- We assume that $F(u) = 1 - exp(-2u)$, exponential.
- Then $\int F(u)du = u + \frac{1}{2}exp(-2u) + C$ is easy to calculate.

The table on the next slide shows the calculated results.

Unscheduled valuation – slide 3

t1	t2	r1	r2	p_{(t1,t2]}, Equation (5)	pi_{(t1,t2],(r1,r2]} Equation (6)
1	2	5	6	1	0.09 %
1	2	4	5	1	0.68 %
1	2	3.5	4	1	1.36 %
1	2	3	3.5	1	3.70 %
1	2	1	3	1	94.15 %
2	3	5	6	1	0.68 %
2	3	4	5	1	5.06 %
2	3	3.5	4	1	10.05 %
2	3	3	3.5	1	27.33 %
2	3	2	3	1	56.77 %
3	3.5	5	6	0.5	2.72 %
3	3.5	4	5	0.5	20.11 %
3	3.5	3.5	4	0.5	39.96 %
3	3.5	3	3.5	0.5	36.79 %
3.5	4	5	6	0.5	7.40 %
3.5	4	4	5	0.5	54.66 %
3.5	4	3.5	4	0.5	36.79 %
4	5	5	6	1	37.38 %
4	5	4	5	1	56.77 %

The blue rows show those parts of the notification pattern that can be taken from the discrete version.

The yellow/pinkish lines show those parts that only the continuous version can deliver.

Unscheduled valuation – slide 4

<u>Take-aways</u>

- When the actuary has built her data infrastructure, models, reports etc., around fixed-period valuations, an unscheduled valuation outside the planned valuation dates is not a trivial task. To develop continuous time models "on the fly" is normally not an option.
- To be prepared for unscheduled valuations, the actuary must have set up her data infrastructure, models, reports etc., in a continuous time framework in advance. Having done that, the unscheduled valuation could feel like just another day at the office.
- Even when using discrete time methods in routine valuations, the provident actuary should base her discrete time notification patterns on a continuous time model. This will guarantee that claim valuations can be done at an arbitrary date, and that notification patterns used at the arbitrary date are consistent with notification patterns used in routine valuations.

Calibration

Calibration requires determination of three components:

1. The risk exposure function W on $[0, \infty)$, indicating the number of risks exposed to claim occurrence at any point in time. Every insurer should know it.
2. The claim rate function $\lambda(dt)$. It can be modelled as constant or with variations, depending on the type of insurance. Getting its level absolutely right is of secondary importance, as we also have θ.
3. The waiting time distribution F. Using a sample of claims with their accident dates and notification dates, estimating F should be an easy exercise.

A good numerical integration tool is also needed. Calibration details are necessarily context-specific.

Claim subcohorts

Every accident year claim cohort could be partitioned into sub-cohorts that we denote by the superscript $[c]$. If all cohorts are subject to the same claim rate function and waiting time distribution, the only thing that changes is that W becomes $W^{[c]}$, and we can calculate risk volumes and (different!) notification patterns per cohort. Possible cohorts:

- Contract year as in Neuhaus (2021).
- Individual contracts, making calibration easy.

Other applications

Other possible applications of Hesselager's model.

- Different patterns per [accident year, contract year] cohort, as in Neuhaus (2021). Should be useful for IFRS 17.
- Being able to generate mutually consistent notification patterns for any discretisation (year, quarter, month). Even with non-square cells, e.g., accident year by notification quarter. Not exploding time in every dimension, reduces the Excel overload.
- Going from notification patterns to payment patterns. It's the same mechanics, with λ a claim cost intensity and F a payment delay distribution.

Thank you

This paper marks my 20th anniversary of teaching at ISEG.
Many thanks to past and present faculty and staff!