Ruin Probabilities in the context of the Winner's Curse

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Winner's curse

- The winner's curse is a tendency for the winning bid in an auction to exceed the intrinsic value or true worth of an item
- Capen, Clapp, Campbell, 1971 - *Competitive bidding in high-risk situations*
- This paper concerned the companies that trade oil

Actuarial context

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- Insurance aggregator sites
- Some UK and US aggregators include offers from more than 100 companies !!!
- Here the lowest price wins (we have then the so-called reversed auction)
- Quite often the price is lower than the level of the expected value of the future costs (including all operational costs and expected profit) !!!
- In fact, this insurance company can compensate the loss by selling other insurance and financial products which very often are not offered in insurance aggregators. To do this efficiently this insurance company needs to estimate the size of the risk related with selling the product sold on the aggregator and estimate possible bankruptcy probability by selling a large volume of unprofitable and undesirable polices !

Actuarial context

Our goal is to create simple model and estimate so-called ruin probability.

Decaying attachment to the insurer

"Probably, this phenomenon is caused by the fact that insurance aggregators emerged in recent years. Previously, the time and effort involved in seeking alternative quotes meant that many individuals would renew with existing providers if the provider had a trusted brand. Today, the ubiquity of aggregator sites advertising and the ease of access to a huge range of quotes means that behaviour is changing and most policyholders are tempted to check the competitiveness of their renewal quotes each year. As a result, even insurers who seek to avoid exposure to aggregator sites are effectively exposed to an auction, as they may only win the business if they effectively underbid the all of the companies on the aggregator. Therefore incorporating winner's curse in a valuation process is so crucial."

General Insurance Research Organization considers
as the one the most important problems last years ...

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- This happens in modern aggregators! When one fills all given details each time the price for the same insurgence product is different!

Value at Risk

Value at Risk measure (VaR) estimates the potential loss for a given $1 - \alpha$ confidence interval for fixed α

Theorem

$$\mathrm{VaR} = \nu - F^{-1}(1 - \sqrt[N]{1-\alpha})$$

Corollary

If $X_i \sim U[\nu - \theta, \nu + \theta]$ dla $i = 1, \dots, N$, then

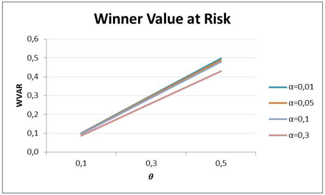
$$\mathrm{VaR} = \theta \left(2 \sqrt[N]{1-\alpha} - 1 \right)$$

Numerical example

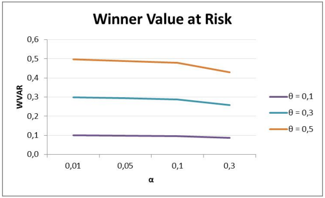
θ \ α	0,01	0,05	0,1	0,3
0,1	0,100	0,098	0,096	0,086
0,3	0,299	0,294	0,287	0,259
0,5	0,498	0,490	0,479	0,431

VaR for $N = 5$ and uniform distribution of F with $\mathrm{U}[\nu - \theta, \nu + \theta]$

Numerical example



Dependence of VaR on θ, $N = 5$



Dependence of VaR on α, $N = 5$

Poisson process of wining the auction

- We assume that an insurance company approaches the bidding with fixed intensity.
- With certain fixed probability it wins the auction
- Thanks to thinning property of a Poisson process the resulting process of getting the insurance contract and hence deriving the premium is a Poisson process
- We assume that this process $M(t)$ has a fixed intensity λ_2

Winner's surplus process

$$U(t) = u + \sum_{j=1}^{M(t)} Y_j - \sum_{i=1}^{N(t)} X_i,$$

where

- The individual premium amounts $\{Y_j\}_{j=1}^{\infty}$
- The arrival claim process $N(t)$ is a renewal process with the times of claim arrivals $\{T_i\}_{i=1}^{\infty}$ and i.i.d. claim sizes $\{X_j\}_{j=1}^{\infty}$. Arrival processes and claim sizes are independent on each other.
- We assume that the interclaim times W_i follow a distribution K_1 with density k_1, and the claim sizes X_i follow a distribution P_1 with density p_1. In the same way, assume that the times between premium arrivals V_j follow a exponential distribution with density $k_2(x) = \lambda_2 e^{-\lambda_2 x}$, and the premium sizes Y_j follow a distribution P_2 with density p_2.

Ruin probability

Our goal is to find so-called ruin probability

$$\psi(u) = P(\tau < \infty \mid U(0) = u)$$

where

$$\tau = \inf\{t \geq 0 : U(t) < 0\}$$

is the ruin time.

ODE

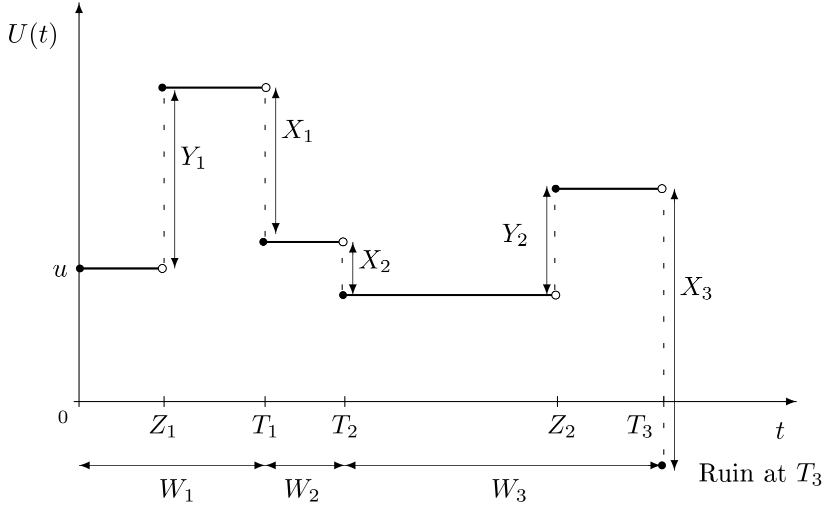
Theorem

$$\begin{aligned}\psi(u) &= P_{-1,2}\int_0^\infty \psi(u+y)p_2(y)dy + \\ &P_{1,-2}\left[\int_0^u \psi(u-x)p_1(x)dx + 1 - P_1(u)\right] + \\ &P_{1,2}\int_0^\infty \left[\int_0^{u+y}\psi(u+y-x)p_1(x)dx + 1 - P_1(u+y)\right]p_2(y)dy\end{aligned}$$

where

$$\begin{aligned}P_{-1,2} &= \int_0^\infty k_2(t)(1-K_1(t))dt \\ P_{1,-2} &= \int_0^\infty k_1(t)(1-K_2(t))dt \\ P_{1,2} &= \int_0^\infty k_1(t)k_2(t)dt\end{aligned}$$

Sparre-Andersen model: transformation



Ruin probability once again

$$\psi(u) = P\left(\exists t \geq 0 : \; \overline{U}(t) < 0\right),$$

where

$$\overline{U}(t) = u + t - \sum_{i=1}^{\overline{N}(t)} X_i,$$

and $\overline{N}$ is a renewal process with kth interarrival time

$$\overline{W}_k = \sum_{l=M(T_{k-1})}^{M(T_k)} Y_l$$

Since $\overline{W}_k$ might have an atom at zero, instead of the original claim sizes it is better to consider the following geometric batch claims:

$$\overline{X}_i = \sum_{j=1}^{L} X_i,$$

where

$$P(L = k) = p^{k-1}(1-p), \qquad k = 1, 2, \ldots$$

with

Exponential case

If we take $k_1(x) = \lambda_1 e^{-\lambda_1 x}$ and $p_1(x) = \beta_1 e^{-\beta_1 x}$ then

$$\overline{W} = \delta_0 \frac{\lambda_1}{\lambda_1 + \lambda_2} + \frac{\lambda_2}{\lambda_1 + \lambda_2} \text{Exponential}\,(\lambda)\,,$$

where

$$\lambda = \frac{\lambda_1 \beta_2}{\lambda_1 + \lambda_2}$$

and

$$f_X(x) = (1-p)\beta_1 e^{-\beta_1(1-p)x},$$

where

$$p = \frac{\lambda_1}{\lambda_1 + \lambda_2}$$

Hence

$$\psi(u) = \frac{\lambda}{\delta} e^{(\delta - \lambda)u}$$

where $\delta = (1-p)\beta_1$.

Erlang case

Assume now that N is a renewal process with interarrival times with Erlang (2) distribution, that is,

$$k_1(x) = \lambda_1 x e^{-\lambda_1 x}.$$

Then

$$\begin{aligned}\overline{W} &= \delta_0 p + a\text{Exponential}\,(\lambda_0) \\ &+ (1-a)\text{Erlang}\,(2, \lambda_0)\end{aligned}$$

and

$$f_X(x) = \delta e^{-\delta x},$$

where

$$p = \left(\frac{\lambda_1}{\lambda_1 + \lambda_2}\right)^2$$

$$a = 2\frac{\lambda_1\lambda_2}{2\lambda_1\lambda_2 + \lambda_2^2}$$

$$\delta = \beta_1(1-p)$$

$$\lambda_0 = \frac{\lambda_1\beta_2}{\lambda_1 + \lambda_2}$$

Erlang case

In this case $\psi(u)$ solves the following IDE:

$$\psi^{''}(u) + c_1\psi^{'}(u) + c_0\psi(u) = c_0\delta \int_0^u \psi(u-y)e^{-\delta y}\, dy + c_0 e^{-\delta u}.$$

where

$$c_0 = \lambda_0^2 \frac{3-5a}{1-a}$$
$$c_0 = -\lambda_0 \frac{2(2a-1)}{1-a}$$

THANK YOU

for Your Attention !!!!!!